INVESTIGATION 2-B

Verifying the Circular Motion Equation

TARGET SKILLS

- Performing and recording
- Analyzing and interpreting
- Communicating results

You have seen the derivation of the equation for circular motion and solved problems by using it. However, it is always hard to accept a theoretical concept until you test it for yourself. In this investigation, you will obtain experimental data for uniform circular motion and compare your data to the theory.

Problem

How well does the equation describe actual experimental results?

Equipment

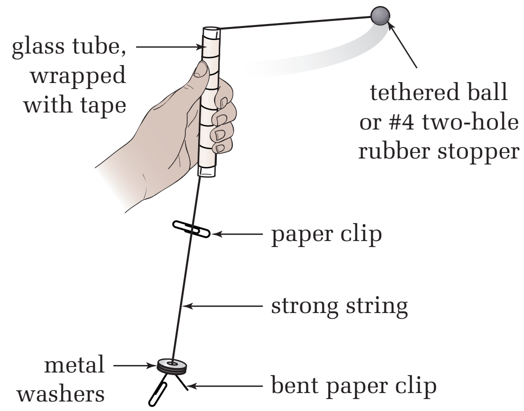
- laboratory balance
- force probeware or stopwatch
- ball on the end of a strong string
- glass tube (15 cm long with fire-polished ends, wrapped in tape)
- metre stick
- 12 metal washers
- tape
- paper clips

CAUTION Wear impact-resistant safety goggles. Also, do not stand close to other people and equipment while doing this activity.

Procedure

Alternative A: Using Traditional Apparatus

1. Measure the mass of the ball.
2. Choose a convenient radius for swinging the ball in a circle. Use the paper clip or tape as a marker, as shown in the diagram at the top of the next column, so you can keep the ball circling within your chosen radius.
3. Measure the mass of one washer.
4. Fasten three washers to the free end of the string, using a bent paper clip to hold them in place. Swing the string at a velocity that will maintain the chosen radius. Measure the time for several revolutions and use it to calculate the period of rotation.



5. Calculate the gravitational force on the washers (weight), which creates tension in the string. This force provides the centripetal force to keep the ball moving on the circular path.
6. Repeat for at least four more radii.

Alternative B: Using Probeware

1. Measure the mass of the ball.
2. Attach the free end of the string to a swivel on a force probe, as shown in the diagram on the next page.
3. Set the software to collect force-time data approximately 50 times per second. Start the ball rotating at constant velocity, keeping the radius at the proper value, and collect data for at least 10 revolutions.

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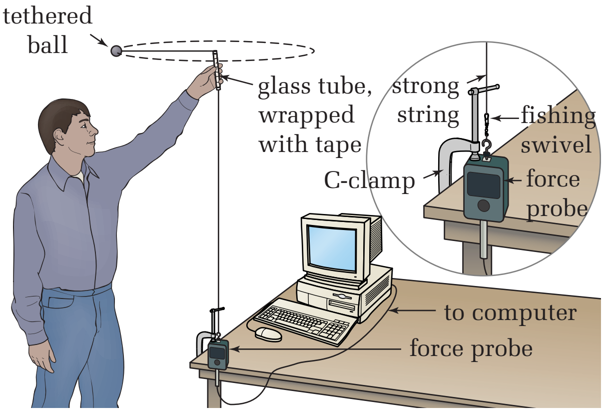
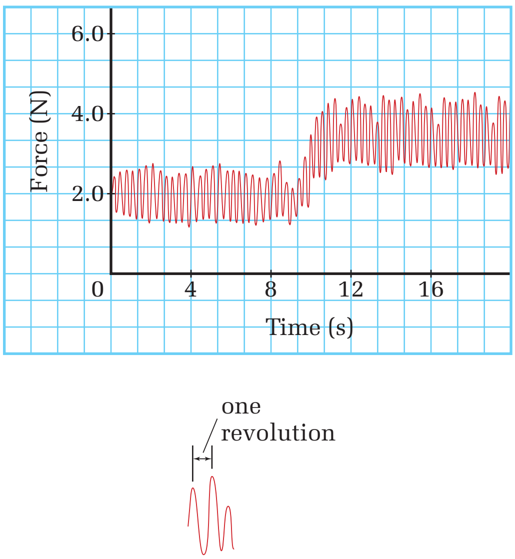


4. Examination of the graph will show regular variations from which you can calculate the period of one revolution, as well as the average force.



5. Repeat for at least five different radii.

Analyze and Conclude

1. For each radius, calculate and record in your data table the velocity of the ball. Use the period and the distance the ball travels in one revolution (the circumference of its circular path).
2. For each radius, calculate and record in your data table $\frac{mv^2}{r}$.
3. Graph F_c against $\frac{mv^2}{r}$. Each radius will produce one data point on your graph.
4. Draw the best-fit line through your data points. How can you tell from the position of the points whether the relationship being tested, $F_c = \frac{mv^2}{r}$, actually describes the data reasonably well?
5. Calculate the slope of the line. What does the slope tell you about the validity of the mathematical relationship?
6. Identify the most likely sources of error in the experiment. That is, what facet of the experiment might have been ignored, even though it could have a significant effect on the results?

Apply and Extend

Based on the experience you have gained in this investigation and the theory that you have learned, answer the following questions about circular motion. Support your answers in each case by describing how you would experimentally determine the answer to the question and how you would use the equations to support your answer.

7. How is the required centripetal force affected when everything else remains the same but the frequency of rotation increases?
8. How is the required centripetal force affected when everything else remains the same but the period of rotation increases?
9. If the radius of the circular path of an object increases and the frequency remains the same, how will the centripetal force change?
10. How can you keep the velocity of the object constant while the radius of the circular path decreases?

Banked Curves

Have you ever wondered why airplanes tilt or bank so much when they turn, as the airplanes in the photograph are doing? Now that you have learned that a centripetal force is required in order to follow a curved path or turn, you probably realize that banking the airplane has something to do with creating a centripetal force. Land vehicles can use friction between the tires and the road surface to obtain a centripetal force, but air friction (or drag) acts opposite to the direction of the motion of the airplane and cannot act perpendicular to the direction of motion. What force could possibly be used to provide a centripetal force for an airplane?

When an airplane is flying straight and horizontally, the design of the wings and the flow of air over them creates a lift force (L) that keeps the airplane in the air, as shown in Figure 2.11. The lift must be equal in magnitude and opposite in direction to the weight of the airplane in order for the airplane to remain on a level path. When an airplane banks, the lift force is still perpendicular to the wings. The vertical component of the lift now must balance the gravitational force, while the horizontal component of the lift provides a centripetal force. The free-body diagram on the right-hand side of Figure 2.11 helps you to see the relationship of the forces more clearly.

Figure 2.10 When an airplane follows a curved path, it must tilt or bank to generate a centripetal force.

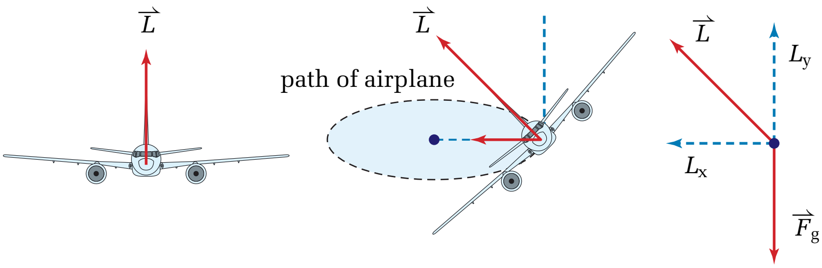


Figure 2.11 When a pilot banks an airplane, the forces of gravity and lift are not balanced. The resultant force is perpendicular to the direction that the airplane is flying, thus creating a centripetal force.

Cars and trucks can use friction as a centripetal force. However, the amount of friction changes with road conditions and can become very small when the roads are icy. As well, friction causes wear and tear on tires and causes them to wear out faster. For these reasons, the engineers who design highways where speeds are high and large centripetal forces are required incorporate another source of a centripetal force — banked curves. Banked curves on a road function in a way that is similar to the banking of airplanes.

Figure 2.12 shows you that the normal force of the road on a car provides a centripetal force when the road is banked, since a normal force is always perpendicular to the road surface.

You can use the following logic to develop an equation relating the angle of banking to the speed of a vehicle rounding a curve. Since an angle is a scalar quantity, omit vector notations and use only magnitudes. Assume that you wanted to know what angle of banking would allow a vehicle to move around a curve with a radius of curvature r at a speed v, without needing any friction to supply part of the centripetal force.

- Since a car does not move in a vertical direction, the vertical component of the normal force must be equal in magnitude to the force of gravity.

$$F_N \cos\theta = F_g$$
$$F_N \cos\theta = mg$$

- The horizontal component of the normal force must supply the centripetal force.

$$F_N \sin\theta = F_c$$
$$F_N \sin\theta = \frac{mv^2}{r}$$

- Divide the second equation by the first and simplify.

$$\frac{\cancel{F_N} \sin\theta}{\cancel{F_N} \cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$
$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{v^2}{r}}{g}$$
$$\tan\theta = \frac{v^2}{rg}$$

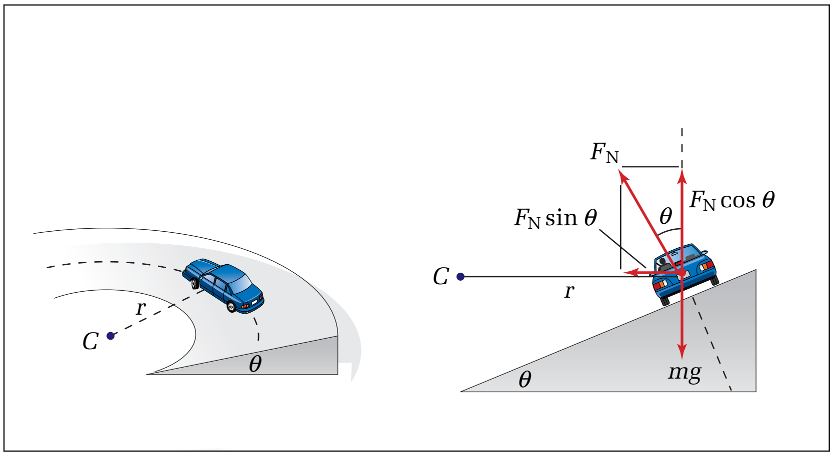
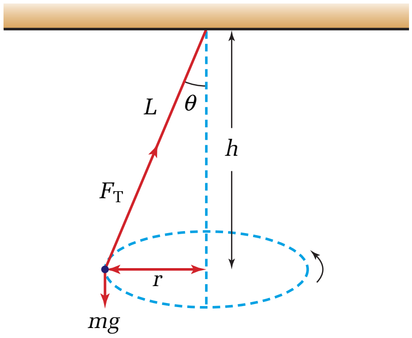


Figure 2.12 When you look at a cross section of a car rounding a curve, you can see that the only two forces in a vertical plane that are acting on the car are the force of gravity and the normal force of the road. The resultant force is horizontal and perpendicular to the direction in which the car is moving. This resultant force supplies a centripetal force that causes the car to follow a curved path.

Notice that the mass of the vehicle does not affect the amount of banking that is needed to drive safely around a curve. A semitrailer and truck could take a curve at the same speed as a motorcycle without relying on friction to supply any of the required centripetal force. Apply what you have learned about banking to the following problems.

Conceptual Problem

- A conical pendulum swings in a circle, as shown in the diagram. Show that the form of the equation relating the angle that the string of the pendulum makes with the vertical to the speed of the pendulum bob is identical to the equation for the banking of curves. The pendulum has a length L, an angle θ with the vertical, a force of tension F_T in the string, a weight mg, and swings in a circular path of radius r. The plane of the circle is a distance h from the ceiling from which the pendulum hangs.



SAMPLE PROBLEM

Banked Curves and Centripetal Force

Canadian Indy racing car driver Paul Tracy set the speed record for time trials at the Michigan International Speedway (MIS) in the year 2000. Tracy averaged 378.11 km/h in the time trials. The ends of the 3 km oval track at MIS are banked at 18.0˚ and the radius of curvature is 382 m.

(a) **At what speed can the cars round the curves without needing to rely on friction to provide a centripetal force?**

(b) **Did Tracy rely on friction for some of his required centripetal force?**

Conceptualize the Problem

- The *normal force* of a *banked curve provides* a *centripetal force* to help cars turn without requiring an excessive amount of friction.
- For a given *radius of curvature* and *angle of banking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.

Identify the Goal

(a) The speed, v, for which the normal force provides exactly the required amount of centripetal force for driving around the curve

(b) Whether Tracy needed friction to provide an additional amount of centripetal force

Identify the Variables and Constant

Known	Implied	Unknown
$r = 382\ \text{m}$	$g = 9.81\ \dfrac{\text{m}}{\text{s}^2}$	v
$\theta = 18.0^\circ$		
$v_{PT} = 378.11\ \dfrac{\text{km}}{\text{h}}$		

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Develop a Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

$$\tan\theta = \frac{v^2}{rg}$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta}$$

Substitute the numerical values and solve.

$$v = \sqrt{(382\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\tan 18.0^\circ)}$$
$$v = \sqrt{1217.61\ \frac{\text{m}^2}{\text{s}^2}}$$
$$v = 34.894\ \frac{\text{m}}{\text{s}}$$
$$v \cong 34.9\ \frac{\text{m}}{\text{s}}$$

(a) A vehicle driving at 34.9 m/s could round the curve without needing any friction for centripetal force.

Convert the velocity in m/s into km/h.

$$v = \left(34.894\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{3600\ \cancel{\text{s}}}{\text{h}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)$$
$$v = 125.619\ \frac{\text{km}}{\text{h}}$$
$$v \cong 126\ \frac{\text{km}}{\text{h}}$$

(b) Tracy was driving three times as fast as the speed of 126 km/h at which the normal force provides the needed centripetal force. Paul had to rely on friction for a large part of the needed centripetal force.

Validate the Solution

An angle of banking of 18° is very large compared to the banking on normal highway curves. You would expect that it was designed for speeds much higher than the highway speed limit. A speed of 126 km/h is higher than highway speed limits.

PRACTICE PROBLEMS

20. An engineer designed a turn on a road so that a 1225 kg car would need 4825 N of centripetal force when travelling around the curve at 72.5 km/h. What is the radius of curvature of the road?

21. A car exits a highway on a ramp that is banked at 15° to the horizontal. The exit ramp has a radius of curvature of 65 m. If the conditions are extremely icy and the driver cannot depend on any friction to help make the turn, at what speed should the driver travel so that the car will not skid off the ramp?

22. An icy curve with a radius of curvature of 175 m is banked at 12°. At what speed must a car travel to ensure that it does not leave the road?

23. An engineer must design a highway curve with a radius of curvature of 155 m that can accommodate cars travelling at 85 km/h. At what angle should the curve be banked?

You have studied just a few examples of circular motion that you observe or experience nearly every day. Although you rarely think about it, you have been experiencing several forms of circular motion every minute of your life. Simply existing on Earth's surface places you in uniform circular motion as Earth rotates. In addition, Earth is revolving around the Sun. In the next chapter, you will apply many of the concepts you have just learned about force and motion to the motion of planets, moons, and stars, as well as to artificial satellites.

2.2 Section Review

1. K/U Define uniform circular motion and describe the type of acceleration that is associated with it.

2. K/U Study the diagram in Figure 2.6 on page 79. Explain what approximation was made in the derivation that requires you to imagine what occurs as the angle becomes smaller and smaller.

3. C What are the benefits of using the concept of centripetal acceleration rather than working on a traditional Cartesian coordinate system?

4. K/U Explain how centripetal force differs from common forces, such as the forces of friction and gravity.

5. K/U If you were swinging a ball on a string around in a circle in a vertical plane, at what point in the path would the string be the most likely to break? Explain why. In what direction would the ball fly when the string broke?

6. C Explain why gravity does *not* affect circular motion in a horizontal plane, and why it *does* affect a similar motion in a vertical plane.

7. C Describe three examples in which different forces are contributing the centripetal force that is causing an object to follow a circular path.

8. MC When airplane pilots make very sharp turns, they are subjected to very large *g* forces. Based on your knowledge of centripetal force, explain why this occurs.

9. C A centrifugal force, if it existed, would be directed radially outward from the centre of a circle during circular motion. Explain why it feels as though you are being thrown outward when you are riding on an amusement park ride that causes you to spin in a circle.

10. K/U On a highway, why are sharp turns banked more steeply than gentle turns? Use vector diagrams to clarify your answer.

11. I Imagine that you are in a car on a major highway. When going around a curve, the car starts to slide sideways down the banking of the curve. Describe conditions that could cause this to happen.

UNIT PROJECT PREP

Parts of your catapult launch mechanism will move in part of a circle. The payload, once launched, will be a projectile.

- How will your launch mechanism apply enough centripetal force to the payload to move it in a circle, while still allowing the payload to be released?
- How will you ensure that the payload is launched at the optimum angle for maximum range?
- What data will you need to gather from a launch to produce the most complete possible analysis of the payload's actual path and flight parameters?